Observations and Orbit of the Double Star Σ 1728=42 *Comæ Berenices.* By M. Otto Struve.

This double star, discovered by my father in 1826, began to attract his particular attention since 1833, when it offered for the first time the appearance of a single star. Since 1833 it has been observed as regularly as possible every year by my father at Dorpat and by myself at Pulkowa, and during the last 40 years has presented three times more the rare phenomenon of an occultation of one star by the other. I beg leave to communicate to the Society the complete list of annual means of our observations on this remarkable system, together with the conclusions to which I have been led lately concerning the orbit. My observations, it must be remarked, are corrected throughout for the effect of systematic errors, determined by measures of artificial double stars; I have also applied to the distances estimated by my father the corrections deduced by himself in a similar way (*Mens. Micr.* p. cliv.).

Dorpat Observations.

	″		°	day
1827·28	e = 0·570	P =	10.9	1
29·40	0·640		11·6	3
33·36	Susp. fig. obs.		350·7	2
34·43	... ...		228·3	1
35·39	Oblonga		191·2	4
36·42	0·430		190·2	3
37·40	0·515		190·8	6
38·40	0·483		191·5	3

Pulkowa Observations.

	″	°	day
40·45	0·550	195·7	3
41·41	0·495	194·5	2
42·40	0·318	193·9	3
45·47	Simplex	...	1
46·40	Susp. fig. obs.	67·0	3
47·42	0·200	15·5	3
48·42	0·267	12·7	3
49·42	0·420	8·6	3
50·39	0·480	11·4	3
51·42	0·488	7·0	4
52·43	0·563	10·9	3
53·40	0·570	10·8	3
54·38	0·600	14·1	1

	″	°	day
55·44	0·620	9·1	2
57·49	0·435	7·7	2
58·44	0·385	8·5	2
59·37	Susp. fig. obs.	4·0	4
61·42	0·435	185·6	2
62·40	0·545	191·6	2
63·44	0·550	189·3	1
64·42	0·513	192·5	3
66·44	0·403	188·5	3
67·47	0·365	193·0	2
68·44	0·215	195·8	2
69·47	Susp. fig. obs.	195·0	1
70·44	Simplex	...	2
71·43	Simplex	...	1
72·42	Oblonga	20·0	1
73·46	0·200	9·0	2
74·41	0·302	9·2	2

The two stars are so very nearly of the same brightness (both 6·0 according to my father) that I never could make out with certainty which of the two was the brighter. This circumstance and the apparent rectilinear motion left, even after three occultations, still some doubts concerning the period of revolution, which could be supposed either of about 13 years or of double that time. The recent observations have decided in favour of the second hypothesis, and thereby I have been enabled to distinguish between the two branches of the orbit, as it is done in the foregoing list by the difference of 180° in the indicated directions. The quite insignificant variations in the observed directions in all instances—when the two stars were seen separated or offered together a distinctly elongated figure—show clearly that the plane of the orbit coincides so nearly with the direction to the Sun that we have not the least trace of a sensible inclination. Consequently the situation of the plane is exactly determined. We have to adopt the inclination $i=90°$, and the angle between the ascending node and the declination circle equal to the mean of all observed directions $=11°·0$.

Thus the number of elements to be deduced from our observations is reduced to five. But this simplification of the problem is more than outweighed by the unfavourable condition, that for the deduction of those five elements we cannot make any use of the observed directions, but must rely entirely upon the measured and estimated distances.

Already in 1866 (*Bulletin de l'Académie de St. Pétersbourg*, t. iii.) I have stated the reasons which led me to the conclusion that the period of revolution ought to be supposed about 25·5

years. This conclusion was then principally based upon the remark that between the two first occultations (1834 and 1845) there had elapsed only 11-12 years; while the next following occultation (1859) happened after an interval of rather more than 14 years. This supposition is completely confirmed by the more recent observations, which show that in correspondence with the first period the latest occultation (1870-71) has happened again after about 11 years. Of course, if there is such an alternation in the periods of visibility, two of them will together form an entire period of revolution. But there is now one feature more come out, which, in corroborating these conclusions, must at once appear most useful for a more accurate deduction of all the other elements. We see that about the time of the two corresponding occultations (1845 and 1871) five years have elapsed between the epochs when the two stars ceased to appear separated till they began to separate again, or between distances from about $+0''.2$ to $-0''.2$, while at the time of the intermediate occultation (1859) three years were sufficient to produce a change of from $0''.4$ in the decrease to $0''.4$ in the increase. The occultation of 1834, corresponding to that of 1859, shows likewise a comparatively rapid transition from the phase of simplicity to larger distances. This remark, combined with the difference in the intervals of the occultations, might necessarily lead us to the conclusion that the real orbit of the system is considerably excentrical and its major axis not much inclined to the direction in which we see the stars.

The general figure of the orbit being thus fixed, we proceeded to express it in numerical values; and after some trials, found that the following elements would tolerably well represent all measured and estimated distances:—

Time of passage through periaster	$T = 1860.0$
Angle in the orbit between periaster and ascending node	$\lambda = 100^\circ$
Major semi-axis	$a = 0''.693$
Excentricity	$e = 0.50$
Mean annual motion	$m = 13^\circ 51'$ or revolution $= 26^y.0$.

Starting from these elements, one of my assistants, M. Dubiago, developed the differential formulæ for distances, and proceeded to the inquiry by what elements, according to the method of least squares, all observations would be best represented. For this purpose it seemed desirable to make up, not only of those observations in which the distance of the two centres had been immediately measured or estimated, but also of those in which, on account of their closeness, the distances had to be derived from the general appearance produced by the two stars together. Accordingly, I examined carefully the expressions used on those occasions in the original manuscripts. This examination, together with the experience won by numerous

other observations, engaged me to fix upon the following distances as the most probable:—

1845·47	70·44	71·44		d = 0″·00
33·36	46·40			= 0·10
34·43	59·37	69·47	72·42	= 0·15
35·39				= 0·25

These estimations can hardly be subject to errors of more than a few hundredth figures of the second decimal, as the whole amount does not exceed 0″·25, and therefore may be regarded as of nearly the same weight as the direct measures and estimations. Then, again, in the list of immediately estimated distances there were two to which I proposed to attribute only half the weight of that of the other observations. These were the observations of 1840 and 1841. It is a curious and not yet sufficiently explained fact, that all distances of double stars measured and estimated by me during the first two years of my residence at Pulkowa, come out by 0″·1 to 0″·2 greater than those observed by my father at Dorpat, and likewise than those observed by me since 1842 at Pulkowa, for which I have determined the constant and systematic errors by means of artificial double stars. The exact value of that constant difference will result from a general discussion of all my micrometrical measures, which I hope I shall soon be able to submit to the astronomical public. But before this is done, I thought it best for the present research to leave these earlier observations unaltered, but at present to give them only half the weight.

Thus our observations offered M. Dubiago 38 equations of condition. Having resolved them by the method of least squares, he found the following elements:—

$$T = 1869{\cdot}92 \pm 0^{y}{\cdot}080$$
$$\lambda = 99^{\circ}{\cdot}11' \pm 0^{\circ}{\cdot}45^{s}{\cdot}6$$
$$a = 0''{\cdot}657 \pm 0''{\cdot}0126$$
$$e = 0{\cdot}480 \pm 0{\cdot}0239$$
$$m = 14^{\circ}0'{\cdot}2 \pm 2'{\cdot}75, \text{ or revolution} = 25^{y}{\cdot}71 \pm 0^{y}{\cdot}084$$

With these elements M. Dubiago computed directly the distances, and got the following differences between observation and computation:—

	O−C ″		O−C ″
1827·28	−0·023	1853.40	−0·038
29·40	+0·014	54·38	−0·028
33·36	−0·070	55·44	0·000
34·43	+0·037	57·49	−0·036
35·39	−0·078	58·44	+0·072
36·42	−0·034	59·37	+0·057

	O−C		O−C
	″		″
37·40	−0·006	61·42	+0·053
38·40	−0·041	62·40	+0·057
40·45	+0·116	63·44	+0·023
41·41	+0·130	64·42	−0·004
42·40	+0·036	66·44	−0·012
45·47	−0·003	67·47	+0·028
46·40	+0·008	68·44	−0·039
47·42	+0·011	69·47	−0·010
48·42	−0·012	70·44	−0·063
49·42	+0·055	71·43	−0·027
50·39	+0·039	72·42	+0·028
51·42	−0·024	73·46	−0·019
52·43	−0·005	74·41	−0·002

From these differences we deduce the probable error of the single annual means=0″·030, or that of a single observed distance =0″·046, certainly a very satisfactory result. The only two distances that differ more than 0″·1 from the computation are the two prospected of the years 1840 and 1841; the sign of their differences and their amount agrees completely with what is indicated by numerous other comparisons. If we had rejected entirely those two distances, the probable error would have been found considerably less; and probably also the general distribution of the sign of the differences would have been yet more satisfactory than it is now.

The elements having been deduced exclusively from my father's and my own observations, it was interesting to examine how the observations of other astronomers would agree with them. For this purpose M. Dubiago has compared also the distances observed by the late Mr. Dawes and by M. Secchi.

Observations of Mr. Dawes.

	Distance.			
	Obs.	Comp.		O−C
	″	″	days	″
1840·74	0·425 est.	0·414	3	+0·011
42·53	Simplex	0·272	1	(−0·272)
43·45	Simplex	0·189	1	(−0·189)
53·09	0·626	0·596	4	+0·030
54·39	0·552	0·628	5	−0·076
60·34	0·2 est.	0·108	1	+0·092
63·25	0·8 est.	0·524	1	−0·024
64·43	0·45 est.	0·517	1	−0·067

Observations of M. Secchi.

	Obs. ″	Comp. ″	days	O−C ″
1856·35	0·45 est.	0·580	2	−0·130
57·37	0·486	0·486	3	0·000
58·39	0·3 est.	0·323	1	−0·023

Evidently for both observers the agreement is likewise quite satisfactory. There is only Dawes's observation of 1842, for which the difference is greater than might be expected. In fact, it might appear strange that Dawes, with his excellent eye, should have said that year the star appeared to him quite round, if the real distance of the centre had been 0″·272, but all conclusions on this account lose their power if we consider that in that year he had only one evening's observations, and that he says expressively that on this occasion the air was not good. The power used by him that night was only of 420, hardly sufficient to show distinctly such small quantities, if the state of the atmosphere was not first rate.

With regard to the directions, the agreement of the different observers is as perfect as can be desired. If we exclude those observations for which the closeness of the two stars did not admit of a direct estimation or measure of the distance, the means for the individual observers are:—

		°
W. Struve P	=	10·9
O. Struve	=	11·1
Dawes	=	12·2
Secchi	=	12·4
Dembowski	=	9·6

and the single observations deviate from these means only of quantities that can be regarded as quite evanescent even at the maximum distance of 0″·65. Future observations, by still more perfect instruments, will, perhaps, decide if the larger discrepancies from those means, indicated by my father and myself at epochs (1834, 1846, and 1859) when we were hardly able to discern a deviation from the circular form, were real or not. A very small error in the centration of the object-glass, so small that its existence would hardly be suspected with single stars, might, on these occasions, have been sufficient to modify considerably our judgment on the directions in which the image appeared elongated. We see also that, amongst the three mentioned observations, that of 1859 is in manifest contradiction with the two others. If the angle observed by my father in 1834, shortly after occultation, was really larger in the middle direction, then it ought to have been found smaller in 1859, half a year before occultation, or vice versâ. Therefore, we can say

that the supposition of a perfect coincidence of the ray of vision with the orbit is as exact as the observations admit of.

The small probable errors deduced by M. Dubiago show that the elements are probably the most accurate of all that until now have been computed for orbits of double stars. This result is greatly due to the favourable circumstance that in this case we could dispose of nearly two complete revolutions. But it is also a remarkable fact that these elements rest entirely upon distances; which until now have acted only a very subordinate part in the determination of similar orbits.

Pulkowa, 1875, *April*.

Elements of Binary Stars. By Dr. W. Doberck.

μ^2 Boötis.

Node = 182° 59′
γ = 44° 26′
λ = 17° 41′
e = 0·6174
P = 290·07 yrs.
T = 1863·51
a = 1″·500.

b Coronæ.

Node = 6° 43′
γ = 29° 40′
λ = 89° 17′
e = 0·7502
P = 843·2 yrs.
T = 1828·91
a = 6″·001.

Col. Cooper's Observatory,
1875, *May*.
